

Programme des Journées de Dynamique

28 au 30 septembre 2022

Mercredi 28 septembre

- **14h Sophie Grivaux : Some new results regarding convergence under \times_q of \times_p invariant measures on the circle.**

For each integer $n \geq 1$, denote by T_n the map $x \mapsto nx \pmod 1$ from the circle group $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ into itself. Let $p, q \geq 2$ be two multiplicatively independent integers. Using Baire Category arguments, we will show that generically, a continuous T_p -invariant probability measure μ on \mathbb{T} is such that $(T_{q^n}\mu)_{n \geq 0}$ does not converge w^* to the Lebesgue measure on \mathbb{T} . This disproves Conjecture (C3) from a 1988 paper by R. Lyons, which is a stronger version of Furstenberg's rigidity conjecture on \times_p and \times_q invariant measures on \mathbb{T} , and complements previous results by Johnson and Rudolph.

The talk will be based on a joint work with Catalin Badea (Lille).

- **15h10 Rafael Potrie : Geometric properties of partially hyperbolicity and applications to measure rigidity.**

I will try to explain a work in progress with A. Eskin and Z. Zhang where we look at the geometry of partially hyperbolic measures in 3-manifolds and try to quantify their non-joint integrability depending on properties of some objects that we call templates (motivated by the work of M. Tsujii and Z. Zhang on exponential mixing). This relates well with recent work on measure rigidity by A. Katz and similar works by S. Alvarez, M. Leguil, D. Obata and B. Santiago which try to extend the techniques of A. Eskin and M. Mirzakhani to the smooth dynamics setting. Time permitting, I will explain some consequences for partially hyperbolic Anosov diffeomorphisms in T3.

- **16h40 Nuria Fagella : Structural stability in complex dynamics.**

We discuss the concept of structural stability in holomorphic families of meromorphic maps, in the spirit of the celebrated theorem of Mañé-Sad-Sullivan and Lyubich in the 1980's for rational maps. We show that, for functions with an essential singularity at infinity, a new type of bifurcation occurs which consists of periodic points disappearing to infinity. Nevertheless, under some finiteness conditions, we show that J-structurally stable maps are open and dense in the appropriate parameter space (of arbitrary dimension). (Joint work with Matthieu Astorg and Anna Miriam Benini)

Jeudi 29 septembre

- **9h Faustin Adiceam : Autour du problème de Danzer et de la construction de forêts denses.**

Le problème de Danzer (1961) pose la question de l'existence d'un ensemble de densité finie (i.e. "ne contenant pas beaucoup de points") intersectant tout corps convexe de volume unité. Il a attiré à lui une somme considérable de travaux regroupant un large spectre des mathématiques modernes (en particulier la théorie ergodique et les systèmes dynamiques). Après avoir présenté quelques-uns d'entre eux, nous nous intéresserons à une approche récente obtenue en relâchant la contrainte de volume. Ceci conduit au problème de la construction de forêts dites denses.

- **10h Frédéric Paulin : Equidistribution partielle de rayons de Farey en courbure négative. / Partial equidistribution of Farey rays in negative curvature**

Dans le fibré unitaire tangent d'une variété riemannienne à courbure strictement négative pincée de volume fini, une feuille instable forte fermée poussée par le flot géodésique s'équidistribue vers la mesure d'entropie maximale. En fixant des familles discrètes de points d'origine géométrique (intersection avec des orbites divergentes du flot géodésique) sur ces feuilles instables et en prenant bien garde à en avoir ni trop ni trop peu (c'est-à-dire avec une densité prescrite), nous montrons qu'ils s'équidistribuent vers une mesure supportée sur une feuille stable faible tronquée. Nous donnons des applications arithmétiques en variant les variétés hyperboliques arithmétiques. Ceci est un travail en commun avec Jouni Parkkonen.

- **11h Julien Trévisan : Convergence in distribution of the error term in the lattice point problem.**

In this talk, we are interested in the following general problem. Let L be a unimodular lattice of \mathbb{R}^d . Let S be a measurable set of finite volume. What is the number $N(S, L)$ of points that belong to both L and S ? When S is sufficiently regular, it can be shown that this number $N(S, L)$ is approximated by $vol(S)$ to within an error $R(S, L)$. In this talk, we are going to present results that precise the behaviour of $R(t * S, L)$, with $t > 0$ going to infinity, in several different situations. Dynamical methods are needed to prove some of the presented results.

- **14h30 Nguyen Thi Dang : Equidistribution of flat periodic tori**

Bowen and Margulis in the 70s proved that closed geodesics on compact hyperbolic surfaces equidistribute towards the measure of maximal entropy. From a homogeneous dynamics point of view, this measure is the quotient of the Haar measure. In a joint work with Jialun Li, we study a higher rank generalization of this homogeneous dynamics problem. In the compact case, what we consider instead of closed geodesics are periodic flat tori of dimension ≥ 2 . I will present our equidistribution formula and sketch the proof. We follow Roblin's counting strategy with a quantitative shortcut due to Gorodnik–Nevo which allow us to deduce the main term and an exponential error term.

- **15h30 Dmitry Kleinbock : Dimension drop conjecture in homogeneous dynamics**

Let (X, μ, T) be an ergodic probability-measure preserving system on a metric space X , and let U be a non-empty open subset of X . Consider the (null) set of points in X whose trajectory misses U . When can one prove that this exceptional set has Hausdorff dimension less than the dimension of μ ? This dimension drop phenomenon has been conjectured for actions on homogeneous spaces, and proved in several special cases, for example when X is compact or has rank one. I will talk about proof of the fairly general case of the conjecture – for arbitrary Ad-diagonalizable flows on irreducible quotients of semisimple Lie groups. Two main ingredients of the proof are effective mixing and the method of integral inequalities for height functions on X . Joint work with Shahriar Mirzadeh.

Vendredi 30 septembre

- **9h Nguyen Bac Dang : Spectre du Laplacien sur le groupe associé à la Basilica et dynamique holomorphe.**

Dans cet exposé, issu d'un travail en commun avec Eric Bedford, Rostislav Grigorchuk et Mikhail Lyubich, je vais expliquer comment on peut comprendre le spectre du Laplacien sur une suite de graphes réguliers qui vont converger vers la Basilica (l'ensemble de Julia de $z^2 - 1$). Pour cela, on va être amené à étudier la dynamique d'une application rationnelle explicite, précisément au comportement sous itération des préimage de droites particulières.

- **10h10 Zhiyuan Zhang : Density of mode-locking property for quasi-periodically forced Arnold circle maps.**

Quasi-periodically forced Arnold circle maps with an irrational frequency ω is given by the formula $(\theta, x) \mapsto (\theta + \omega, x + \tau + \frac{\alpha}{2\pi} \sin(2\pi x) + \beta q(\theta) \pmod{1})$ where θ, x belongs to \mathbb{R}/\mathbb{Z} , q is a continuous function on the circle, $\tau, \beta \in \mathbb{R}$ and $\alpha \in (0, 1)$.

In a joint work with Jian Wang, we have shown that the mode-locking region of the family of quasi-periodically forced Arnold circle maps with a topologically generic forcing function is dense. This gives a rigorous verification of certain numerical observations by physicists.

- **11h40 Anna Florio : Dynamiques universelles au sein des flots d'Euler stationnaires 3-dimensionnels.**

La compréhension de la complexité dynamique d'un fluide idéal a motivé de nombreux travaux. Selon la vision d'Arnold, les flots d'Euler stationnaires devraient présenter des dynamiques aussi complexes que celles observées en mécanique céleste. Avec Pierre Berger et Daniel Peralta-Salas, nous validons la vision d'Arnold en démontrant l'existence d'un ensemble localement dense de solutions stationnaires de l'équation d'Euler dans \mathbb{R}^3 formé par des champs de vecteurs universels. Nous introduisons pour cela de nouvelles méthodes perturbatives au sein des champs de vecteurs Beltrami permettant d'importer des outils de la théorie des bifurcations.

- **14h Danijela Damjanovic : KAM rigidity for some parabolic abelian actions.**

We show certain form of local rigidity for affine abelian actions on tori, with parabolic linear part. This is joint work with Fayad and Saprykina.

- **15h10 Gabriella Pinzari : On the co-existence of stable and whiskered tori in the three-body problem.**

We will discuss that, using two different systems of canonical coordinates, it is possible to obtain an equilibrium point for the Hamiltonian of the three-body problem which is simultaneously elliptic and hyperbolic. We shall manage to prove the existence of whiskered tori in a sea of maximal ones. The work is still in progress, and some quantitative estimates have been obtained by Dr Xiang Liu, during his master thesis at the University of Padua.

- **16h40 Dmitry Dolgopyat : Exponential Mixing implies Bernoulli.** Exponential mixing is perhaps the strongest chaotic property of dynamical systems. After discussing some recent results concerning statistical properties of exponentially mixing systems we present a proof of the theorem proven in a joint work with Adam Kanigowski and Federico Rodriguez Hertz and saying that a system enjoying exponential mixing with respect to a smooth measure is isomorphic to a Bernoulli shift.