

# Journées de Dynamique

10 et 11 Octobre 2024

UPCité, Amphithéâtre Turing bâtiment Sophie Germain

**Federica Fanoni** (CNRS, LAMA) *From curve graphs to fine curve graphs, and back*

Fine curve graphs have been introduced by Bowden, Hensel and Webb to study homeomorphism and diffeomorphism groups of closed surfaces. A main tool in their work is the fact that fine curve graphs can be approximated by curve graphs of surfaces with punctures. I will talk about joint work with Sebastian Hensel, where we study to which extent the boundary of the fine curve graph can be approximated via curve graphs of surfaces with punctures. I will also show how fine curve graph techniques can be used to construct a parabolic isometry of a graph of curves of an infinite-type surface.

**Isabelle Liousse** (Univ. Lille) *Réversibilité et échanges d'intervalles*

Un élément d'un groupe  $G$  est dit réversible s'il est conjugué dans  $G$  à son inverse.

Dans cet exposé, nous nous intéresserons aux dynamiques réversibles en dimension 1 et plus précisément aux échanges d'intervalles réversibles.

Nous expliquerons pourquoi les propriétés dynamiques d'un échange d'intervalles réversible impliquent l'existence d'un réverseur d'ordre fini et indiquerons quelques conséquences de ce résultat établi en collaboration Nancy Guelman.

**Matteo Ruggiero** (Univ. Paris Cité) *On the Dynamical Manin-Mumford problem for plane polynomial endomorphisms*

The Dynamical Manin-Mumford (DMM) problem is a dynamical question inspired by classical results from arithmetic geometry.

In the case of a polynomial endomorphism  $f$  of the complex plane (of degree  $d \geq 2$ ), the problem consists in determining if an algebraic curve  $C$  containing infinitely many preperiodic points for  $f$  must be itself preperiodic.

While this is not the case in general, in a joint work with Romain Dujardin and Charles Favre we prove that the DMM problem has a positive answer when  $f$  extends to a regular endomorphism of the projective plane, under the additional condition that the action of  $f$  at the line at infinity doesn't have periodic superattracting points.

The proof relies on techniques coming from arithmetic geometry, as well as analytic dynamics both over  $\mathbb{C}$  and over non-archimedean fields.

**Martin Leguil** (Ecole Polytechnique) *Rigidity for conjugacies of dissipative Anosov flows in dimension 3*

In a joint work with Andrey Gogolev and Federico Rodriguez Hertz, we study when two transitive Anosov flows  $X, Y$  in dimension 3 which are topologically conjugated are actually smoothly conjugated. By the work of de la Llave, Marco, Moriyón and Pollicott from the late 80s, a necessary and sufficient condition for that to happen is that stable and unstable eigenvalues at corresponding periodic points match. In our work we show that for generic transitive Anosov flows  $X, Y$  in dimension 3, the latter condition is already implied by the existence of a topological conjugacy; in particular, the conjugacy is smooth, unless the conjugacy swaps positive and negative SRB measures of the two flows. This complements a recent work of Gogolev and Rodriguez Hertz in the volume preserving case. I will try to explain how this rigidity problem is connected

with other notions, in particular, the Foulon-Hasselblatt cocycle, the so-called templates introduced by Tsujii and Zhang to study the regularity of stable and unstable distributions, and a positive proportion Livschits Theorem recently shown by Dilsavor and Reber.

**Corentin Fierobe** (Univ. Roma 3) *On the existence of periodic invariant curves for analytic families of twist-maps and billiards*

Famous KAM results state the persistence of invariant curves for small perturbations of discrete integrable dynamical systems. These curves constitute a large set (i.e. of positive measure) of all curves in the initial system, and on each one of them the system is conjugated to a rotation of Diophantine rotation number. More recent results about so-called integrable billiards and Birkhoffs conjecture underline the importance of invariant curves on which the dynamic is conjugated to a rational rotation : it appears that such curves capture the specificities of a system and are important to study rigidity questions. In this talk, I will present a result obtained jointly with Alfonso Sorrentino on the persistence of such curves in analytic families of twist maps, and which can notably be applied to different billiard models. It extends a result by Arnaud, Massetti and Sorrentino.

**Françoise Pène** (Univ. Bretagne Occidentale) *Stochastic behaviour of chaotic billiards*

We consider a point particle (with random initial position and velocity) evolving in a deterministic chaotic billiard (such as the Sinai billiard in the torus, billiards with cusps, the periodic Lorentz gas with finite or infinite horizon, etc). The initial randomness combined with the hyperbolicity of the dynamics gives rise to interesting stochastic properties. The aim of this talk is to present probabilistic limit theorems in this context.

**Andrea Venturelli** (Univ. Avignon) *On a question of Newtonian mechanics asked by Mark Levi*

In 2003, Mark Levi asked the following question : given a mechanical newtonian system on the plane  $\mathbb{R}^2$ , governed by a potential  $U$ , what can be said on  $U$  if we know that each level curve of  $U$  can be parametrized so that it is a solution of the associated newtonian system? A classical exemple of a potential satisfying this assumption is a potential with a radial symmetry. It is natural to ask if there are others exemples. We will see that the answer depends on some additional assumptions about  $U$ . Specifically, we will see that if  $U$  is real analytic then  $U$  is necessarily radial. If  $U$  is assumed to be smooth and  $\text{Crit}(U)$  is supposed to be totally path disconnected, the conclusion is still the same. But without this assumption on the set of critical points, there are exemples of non radial smooth potentials  $U$  such that each level curve of  $U$  can be parametrized so that it is a solution of the associated newtonian system. This problem is related to an evolution parabolic equation on the set of convex curves on  $\mathbb{R}^2$  called inverse curvature flow.

It is a joint work with Philippe Bolle and Marco Mazzucchelli.

**Béatrice de Tilière** (Univ. Paris Dauphine) *La récurrence dSKP : aspects combinatoires et systèmes géométriques*

le sujet de cet exposé est la récurrence discrète Schwarzienne de Kadomtsev-Petviashvili (dSKP). Dans un premier temps, nous allons démontrer une expression explicite pour sa solution en fonctions des données initiales ; plus précisément, nous montrerons que la solution est le quotient de deux fonctions de partition associées à un modèle de dimères orientés. Dans chacune des fonctions de partition, il y a des annulations, et nous démontrerons une expression alternative, sans annulation, qui implique des arbres et des forêts. En dehors de son intérêt combinatoire, cette expression est utilisée pour montrer des résultats de singularités pour la récurrence. Ensuite, nous expliquerons comment l'équation dSKP apparaît dans de nombreux systèmes de géométrie

discrète tels que : les fonctions holomorphes discrètes, le recoupage de polygones, le pentagram map. Nous exhiberons les implications de nos résultats sur ces systèmes. Il s'agit de travaux en collaboration avec Niklas Affolter (TU Vienne) et Paul Melotti (UP Saclay).

**Laurent Bartholdi** (Saarland University) *Automatic actions*

I will present a general notion of automatic action, based on Büchi automata, and show how it unifies a large number of subclasses, in particular the automatic groups by Cannon, Thurston et al.; the transducer groups by Aleshin, Grigorchuk, Sushchansky, Sidki et al.; substitutional subshifts; and complex dynamics.

I will present some algorithms for these groups, and in particular show under an extra condition (boundedness) that their orbit relation is computable and regular. This will have strong decidability consequences, such as that the order problem, aperiodicity, minimality, etc. for automatic transformations is decidable.

I will detail applications to symbolic dynamics : in particular, it is decidable whether a substitutional subshift is aperiodic, minimal, topologically transitive, etc.; and to complex dynamics : it is decidable e.g. if a Julia set is a Sierpinski carpet.

**Fernando Argenti** (Univ. Zürich)

**Francisco Arana Herrera** (Univ. of Maryland) *Weak mixing rational billiards*

We give a complete classification of the rational polygons whose billiard flow is weak mixing in almost every direction, proving a longstanding conjecture of Gutkin. This is joint work with Jon Chaika and Giovanni Forni.